Comparison Law for Christoffel functions

by

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Let G be a bounded simply-connected domain in the complex plane $\mathbb{C}$, with boundary Γ. The Bergman polynomials $\{p_n(z)\}_{n=0}^{\infty}$ of G are defined by

$$\int p_n(z)\overline{p_m(z)}\,dA(z) = \delta_{m,n}, \quad p_n(z) = \lambda_n z^n + \dots, \quad \lambda_n > 0,$$

where $dA(z)$ denotes the area measure on G.

Let Φ be the conformal map $\overline{\mathbb{C}} \setminus \overline{G} \to \{w : |w| > 1\}$, such that

$$\Phi(z) = \gamma z + \gamma_0 + \gamma_1 \frac{1}{z} + \cdots, \quad \gamma > 0.$$

Then, the normalised Faber polynomials $\{f_n(z)\}_{n=0}^{\infty}$ for G are defined as the polynomial parts of $\Phi'(z)\Phi^n(z)$, normalised so that

$$f_n(z) = \sqrt{\frac{n+1}{\pi}}\gamma^{n+1} z^n + \cdots.$$

Under the assumption that Γ is a quasi-conformal curve we establish the (sharp in both ends) comparison law

$$(1 - \|C\|^2) \sum_{j=0}^{n} |p_j(z)|^2 \le \sum_{j=0}^{n} |f_j(z)|^2 \le \sum_{j=0}^{n} |p_j(z)|^2, \quad z \in \mathbb{C},$$

where $\|\cdot\|$ denotes the $2-$norm of a bounded linear operator in l_2 and C (with $\|C\| < 1$) is the Grunsky matrix associated with Γ.

The purpose of the talk is to show how the comparison law can be applied in order to yield asymptotics for the Bergman and Faber polynomials and the Christoffel functions, defined by

$$\lambda_n(z) := 1 / \sum_{j=0}^{n} |p_j(z)|^2, \quad z \in \mathbb{C}.$$